

**CHRISTIAN SOCIAL SERVICES COMMISSION (CSSC)
NORTHERN ZONE JOINT EXAMINATIONS SYNDICATE (NZ-JES)**



FORM SIX PRE – NATIONAL EXAMINATION 2026

142/1

ADVANCED MATHEMATICS 1

Time: 3 Hours

Wednesday, 25th February 2026 a.m

Instructions:

1. This paper consists of **ten (10)** questions each carrying **ten (10)** marks
2. Answer **all** questions.
3. NECTA Mathematical tables and non – programmable calculators may be used.
4. The work done in each question should be **shown clearly** in the answer space(s) provided.
5. Cellular phones and any unauthorized materials are **not** allowed in the examination room.
6. Write your **Examination number** on every page of your answer booklet(s).

1. By using a non-programmable calculator:

(a) Evaluate $\sqrt[3]{\left(\frac{\pi \tan 30^\circ}{3.414}\right) \left(\frac{e^{4.617} + \cos^{-1}(\ln 0.65)}{\sqrt[5]{e^3}}\right)}$, correct to 6 decimal places

(b) Find $\arg\left(\frac{1}{z_4}\right)$ where $z = 3+4i$, correct to three significant figures.

(c) Evaluate $\int_{0.1}^{0.5} \frac{40}{\pi} \cos\left(\frac{\pi t}{2}\right) dt$, correct to four decimal places.

2. (a) Prove that $\frac{1 + \sinh 2x + \cosh 2x}{1 - \sinh 2x - \cosh 2x} = -\coth x$

(b) Given that $\tanh^{-1}u + \tanh^{-1}v = \frac{1}{2} \ln 5$, prove that $v = \frac{2-3u}{3-2u}$

(c) If $\sec \theta = \cosh u$, with $u > 0$ and $0 < \theta < \frac{\pi}{2}$, express

(i) $\tan \theta$ (ii) $\frac{d\theta}{du}$ in terms of u

Hence, or otherwise, evaluate $\int_0^\infty \frac{1}{\cosh u} du$

3. Two Godowns G_1 and G_2 have grain capacity of 100 quintals and 50 quintals respectively. They supply to three ration shops S_1 , S_2 and S_3 , whose requirements are 60, 50 and 40 quintals respectively. The costs of transportation per quintal from the Godowns to the shops are given in the following table:

Transportation cost per quintal (in Tshs)		
Godowns	Shops	Shipping cost per quintal
G_1	S_1	600
G_1	S_2	300
G_2	S_1	400
G_2	S_2	200
G_1	S_3	250
G_2	S_3	300

(i) How the supplies should be transported in order that the transportation cost is minimum

(ii) What is the overall minimum cost?

4. (a) Derive the formula of calculating mean and standard deviation by coding method.
 (b) The following table gives the frequency distribution of the intelligence quotient X of 500 scholars.

X	130–133	126–129	122–125	118–121	114–117	110–113	106–109
f	6	10	18	28	39	56	75

102–105	98–101	94–97	90–93	86–89	82–85
92	71	49	32	19	5

Find: (i) Standard deviation by coding method using assumed mean = 107.5
 (ii) Semi-Interdecile range

5. (a) Using the properties of sets, simplify the following expression:

$$(A \cap B \cap C) \cup (A' \cap B \cap C) \cup B' \cup C'$$

- (b) Describe the following sets by tabular method:

(i) $\{x: x^3 + 1 = 0, x \in N\}$

(ii) The set of all letters in the word TRIGONOMETRY.

- (c) In a competition a school awarded medals in different categories. 36 Medals in Dance, 12 Medals in Dramatics, 18 Medals in Music's. If these Medals went to a total of 45 persons and only four persons who got Medals in all of the three categories. How many received Medal in exactly two of these categories.

6. (a) The function f and g are defined by $f: x \rightarrow \frac{x+1}{5}, x \in \mathbb{R}$ and $g: x \rightarrow e^x, x \in \mathbb{R}$

(i) Solve $f \circ g(x) = 17$

(ii) State the range of g .

- (b) The graph of $y = \frac{ax^2 + bx + x}{x^2 + qx + r}$ has the lines $y = 2, x = 1$ and $x = 3$ as asymptotes

and a turning point at $(0, 1)$.

(i) Find the constants a, b, c, q and r .

(ii) Show that the graph has a second turning point.

(iii) Sketch the graph showing clearly its turning point and behavior as it approaches the asymptote.

(iv) State its domain and range.

7. (a) (i) Derive the Secant formula for approximating the roots of the function $f(x) = 0$.

(ii) Use the formula obtained in 7(a)(i) above to find the solution of $\sin x + xe^x$ between -3 and -4 , correct to four decimal places. Perform only three iterations.

- (b) Apply both trapezium and Simpson's rule to estimate the area of a quadrant of a circle of radius 8 cm by dividing it into eight intervals. Hence, use the better of these results to find an approximate value of π .

8. (a) Q is the foot of the perpendicular from the point $P(-3,2)$ onto a line AB whose equation $3x+4y=12$. Find the equations to the bisectors of the angle between the lines PQ and AB .
- (b) A circle with Centre P and radius r touches externally both the circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 6x + 8 = 0$. Prove that the x -coordinate of P is $\frac{1}{3}r + 2$, and that P lies on the curve $y^2 = 8(x-1)(x-2)$.
9. (a) Evaluate the following integral $\int e^{4x} \sqrt{1 + e^{2x}} dx$
(**Hint:** Use trigonometric substitution and leave your answer without trig notation)
- (b) Evaluate $\int \sqrt{\frac{x-2}{5-x}} dx$
- (c) Find the area of the region in the first quadrant enclosed by the x -axis, the line $x = \sqrt{3}y$ and the circle $x^2 + y^2 = 4$.
10. (a) If $x = a \sin t - b \cos t$ and $y = a \cos t + b \sin t$, show the $\frac{d^2y}{dx^2} = -\frac{x^2 + y^2}{y^2}$
- (b) If $z(x + y) = x^2 + y^2$, prove that $\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4 \left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$
- (c) The length x of a rectangle is decreasing at the rate of 5 cm / minute and the width y is increasing at the rate of 4 cm / minute . When $x = 8 \text{ cm}$ and $y = 6 \text{ cm}$.

Find the rates of change of;

- The perimeter and
- The area of the rectangle.